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Open and closed string interpretation of SUSY CFT's on branes with boundaries

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Abstract: We consider certain supersymmetric configurations of intersecting branes and branes ending on branes and analyze the duality between their open and closed string interpretation. The examples we study are chosen such that we have the lower dimensional brane realizing an $n+1$ dimensional conformal field theory on its worldvolume and the higher dimensional one introducing a conformal boundary. We also consider two CFTs, possibly with different central charges, interacting along a common conformal boundary. We show with a probe calculation that the dual closed string description is in terms of gravity in an AdS_{n+2} bulk with an AdS_{n+1} defect or two different AdS_{n+2} spaces joined along a defect. We also comment briefly on the expected back-reaction.

Keywords: Field Theories in Higher Dimensions, Brane Dynamics in Gauge Theories, AdS-CFT Correspondance.

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Contents

1. Introduction

AdS/CFT duality can be understood as the statement that a given brane configuration in string theory has two equivalent descriptions, one in terms of open and one in terms of closed strings. The examples in the original work of [1] are branes that in a certain decoupling limit realize a supersymmetric $n+1$ dimensional conformal field theory on their worldvolume, the open string description. In the dual language, the theory is described in terms of closed strings propagating in the near-horizon geometry of the corresponding black brane solution. The conformal invariance of the field theory is reflected in the fact that the near horizon geometry is AdS_{n+2}.

In [2] it was argued that the dual description of a brane with an AdS_{n+1} inside that AdS_{n+2}, is a conformal field theory with boundary or, more generally, with a codimension one defect. In order to verify this claim we study supersymmetric configurations in string theory, with branes intersecting in such a way that the lower dimensional one is one of the basic CFT examples of [1], while the higher dimensional one appears as a supersymmetric codimension one defect in this CFT, such that the lower dimensional brane can end. In particular we discuss the D3-brane ending on 5-branes in IIB string theory, D1 D5 black strings ending on D3-branes and M2-branes ending on M5-branes.

The choice of examples is motivated by the requirement that in the decoupling limit the open strings describe a supersymmetric CFT on a manifold with boundary, that is one half of our duality conjecture. In order to establish that the dual closed string description involves AdS_{n+1} branes in AdS_{n+2} we show that in the probe approximation the higher dimensional brane indeed spans an AdS worldvolume in the near horizon geometry of the lower-dimensional brane.

This scenario should be contrasted with the case of a localizing Minkowski brane. It has been believed for a while that the standard Minkowski$_{d+1}$ brane is dual to introducing a UV cutoff in the field theory, breaking the conformal group from SO$(d+1,2)$ to the $d+1$ dimensional Lorentz group preserved by the brane. This cutoff does not correspond to any known field theory cutoff, rather one should view the supergravity description as the definition of this new cutoff with somewhat unusual properties. Therefore quantitative comparison is difficult; however many qualitative features can be explained nicely, see e.g. [3, 4]. In the AdS case we impose a spatial instead of a momentum cutoff, yielding a dual theory with a well-defined lagrangian description.

In the next section, we review the set-up and duality of [2]. In section 3, we give some general arguments about how a probe 5-brane is embedded in the near horizon geometry created by the D3-branes. In section 4, we generalize this result to M2-branes ending on M5-branes and black strings ending on D3s. For the latter one we compare with the recent probe calculation of Bachas and Petropoulos [5]. In section 5, we perform a Born-Infeld probe calculation for the 4d case, verifying the results obtained in section 3. In section 6, we consider the theory beyond the probe approximation and study how the interplay of bulk versus boundary modes of the CFT is reflected in the gravity fluctuations. We conclude in the final section.

2. The supergravity dual for a CFT on the half-plane

The basic observation giving rise to the duality of [2] is the embedding of the AdS_{d+1} brane in AdS_{d+2}, as it is depicted in figure 1. The big cylinder represents the AdS_{d+2}. The AdS_{d+1} brane has the same global time as the big AdS, that is: it does not move. It ends at the equator of the S^d that is the boundary of the AdS_{d+2}, effectively cutting

JHEP06(2001)063

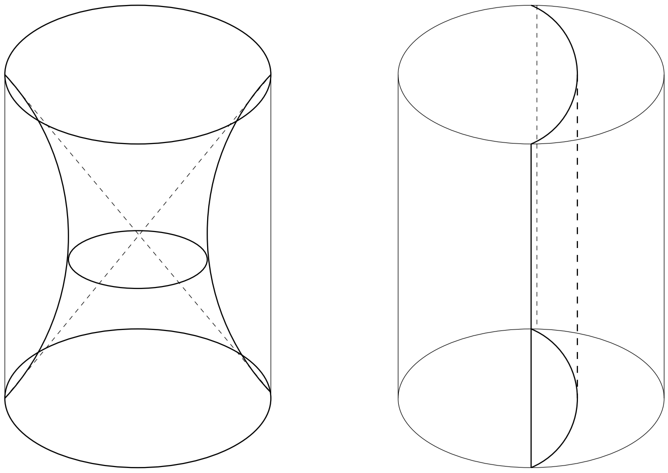

Figure 1: Global embedding of the dS_2 and AdS_2 branes in AdS_3 respectively. The dS case is only included for comparison.

the boundary in half. As we increase the tension the brane curves more and more into the bulk, approaching the true boundary. It nevertheless always ends on the equator of the sphere. For a positive tension brane the part of the cylinder to the right of the brane in figure 1 is removed, including half of the "true" boundary. Gravity in the remaining bulk space-time should have a holographic description in terms of the CFT on the disk plus a theory living on the brane. Appealing to AdS_{d+1} holography one can reduce the description further to a CFT living on the disk with a boundary action living on the common boundary of the disk that remained of the "true" boundary and the brane, basically specifying the boundary conditions for the CFT. Observing that the $\mathrm{SO}(d,2)$ symmetry group preserved by the AdS_{d+1} brane is precisely the subgroup of the conformal group leaving a boundary invariant [6], one is led to believe that the holographic dual of the gravity set-up we described is indeed the original CFT together with a set of conformal boundary conditions specified by the properties of the brane.

If one is looking at the Poincaré patch of the above solution only, the original dual CFT lives on Minkowski space instead of the sphere, and after introducing the AdS brane in the bulk, similarly the boundary CFT will now live on half of Minkowski space, the "upper half plane". Again the boundary conditions reflect the properties of the brane. If one does not impose an orbifold condition relating the independent fluctuations in the two halves of space-time, one ends up with 2 CFTs, each defined on half of Minkowski space, interacting along their common boundary. Or in other words, a CFT living on all of space with a codimension one conformal defect. In general one can couple two different CFTs even with different central charge along a boundary in this fashion by having the background curvature of the AdS_{d+2} jump across the brane. The string theory setup we are describing for non-zero q gives a supersymmetric realization of such a system.

JHEP06(2001)063

3. The probe picture

3.1 The embedding from AdS/CFT

In the introduction we argued that intersecting D3 D5-branes or similar set-ups that are stringy realizations of a conformal field theory on a manifold with boundary should have a near horizon geometry that looks like a warped product of AdS_4 with a gravity localizing warp factor times a compact internal space. In this section we will show that when neglecting the backreaction of the D5-branes on the geometry, one indeed finds that they wrap an equatorial S^2 inside the S^5 and live on an AdS_4 inside the AdS_5. The AdS_4 can be made arbitrarily flat by having some of the D3-branes end on the D5-brane. Let us start with the case of intersecting branes. That is we have branes along

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D 5	x	x	x	x	x	x	o	o	o	o
D 3	x	x	x	o	o	o	x	o	o	o

The near horizon geometry of the N D3-branes is $AdS_5 \times S^5$. When the metric is written as

$$ds^2 = L^2 \left(-\frac{du^2}{u^2} + \frac{u^2}{k_N^2}(dt^2 - dx^2 - dy^2 - dz^2) + ds^2_{S5} \right) , \tag{3.1}$$

where we define k_N as

$$k_N = \sqrt{4\pi g_s N} \tag{3.2}$$

and the curvature length is given by

$$L^2 = l_s^2 \sqrt{4\pi g_s N} = k_N l_s^2 , \tag{3.3}$$

one can make the following identifications [1] between the coordinates of the near horizon region and the coordinates describing the flat embedding coordinates of the branes:

$$\begin{aligned} u^2 &\to \frac{1}{l_s^4}(x_3^2 + x_4^2 + x_5^2 + x_7^2 + x_8^2 + x_9^2) \\ x &\to x_6 \\ t, y, z &\to x_0, x_1, x_2 \end{aligned} \tag{3.4}$$

and the S^5 is the sphere of radius L surrounding the D3-brane,

$$S^5 : \; x_3^2 + x_4^2 + x_5^2 + x_7^2 + x_8^2 + x_9^2 = L^2 . \tag{3.5}$$

Of course one can always change coordinates to $u = k_N v$, such that the metric takes the standard AdS form

$$ds^2 = L^2 \left(-\frac{dv^2}{v^2} + v^2(dt^2 - dx^2 - dy^2 - dz^2) + ds^2_{S5} \right) . \tag{3.6}$$

Now if we add a SUSY probe D5-brane, we know it will span the wordvolume given by

$$x_6 = x_7 = x_8 = x_9 = 0\,. \tag{3.7}$$

That is, it is wrapping the radius L S^2 inside the S^5 given by

$$x_3^2 + x_4^2 + x_5^2 = L^2 \tag{3.8}$$

and is stretching along the surface $x = 0$ inside the AdS_5. The apparent tachyonic instability in fact is above the Breitenlohner-Freedman bound [7], as we will show later. Note that the induced metric on any surface of the form

$$x = \frac{C}{v} \tag{3.9}$$

is an AdS_4 written in the same coordinate system with curvature length

$$l^2 = L^2(1 + C^2)\,. \tag{3.10}$$

When transformed into coordinates covering global AdS, one again sees this hypersurface is the one described in figure 1, where C is the minimal distance between the AdS brane and the center of the cylinder, see [2, 5]. For D3-branes intersecting the D5, the D5-brane worldvolume is the straight (that is, in figure 1 it just cuts straight through the middle of AdS) but maximally curved AdS_4 with curvature radius $l = L$.

Of course this is just the probe approximation. Ultimately, one has to account for the back reaction. Before we do this, it is also of interest to study the case where a large number q of the N D3-branes end on the D5-brane. It will turn out this generates a large separation between l and L, corresonding to nonzero C.

So now we consider the situation where q of the N D3-branes end on the D5-brane. Obviously this requires $q \leq N$. The branes now bend as in [8]. The bending is the solution to a minimal area Laplace problem [8]. As a reaction to being pulled on by the D3-branes, the D5-brane position along x_6 becomes a function of $r^2 = x_3^2 + x_4^2 + x_5^2$. Since this is codimension 3, the bending is of the form

$$x_6 = l_s^2 \frac{\tilde{C}}{r}\,. \tag{3.11}$$

Using that along $x_7 = x_8 = x_9 = 0$ one just gets $r = l_s^2 u$, (3.11) translates into

$$x = \frac{\tilde{C}}{u} = \frac{\tilde{C}}{k_N v} = \frac{C}{v} \tag{3.12}$$

and the bent D5-brane now once more lives along an AdS_4 subspace of AdS_5, this time given by nonzero C. In order to determine C, note that $\tilde{C} = Ck_N$ is proportional to the ratio of the tension of the brane that is pulling to the tension of the brane that is being pulled. For N D3 branes pulling on one side and $N-q$ D3-brane pulling on the other side of M D5-branes we therefore get

$$C = \alpha \frac{q T_{D3}}{M T_{D5}} \frac{1}{l_s^2 k_N} \tag{3.13}$$

where α contains the factors of 2 and π and will be determined later. So we see that for large q/k_N, we get a brane with large curvature radius.

4. Other examples of the geometry of intersecting branes

Although we focus on the D3-D5-brane system, which can result in localized four-dimensional gravity, a similar analysis can be applied to other systems of branes. We describe a few below. One example has actually already been studied [5] and is consistent with our treatment. In other cases, we learn about the expected supergravity solution for other string constructions.

4.1 6d black string with RR charges

We first consider a 6d black string ending on a D3-brane. The black string can either be made out of D1 D5 strings or F1 NS5 strings, where the Q_5 5-branes are always wrapped on the internal 4 manifold M_4 that one used to compactify to 6 dimensions. The role of the D5-brane from above is now played by a D3-brane. Note that it is only the Q_1 strings that can actually end on the D3-brane; the 5-brane part of the black string is not allowed to do so since the 5-brane wraps the internal manifold, while the D3-branes are a point on the internal manifold. So this time we have Q_1 strings and Q_5 5-branes on one side of M D3-branes and $Q_1 - q$ and Q_5 on the other side. In the D1 D5 system the 3d radius of curvature L^2 goes like

$$L^2 = g_{s,6d} \sqrt{Q_1 Q_5} l_s^2 \tag{4.1}$$

so it will jump by q units as above. A string ending on a 3-brane is also codimension 3, so the analysis of the previous section goes through unchanged, we end up with an $AdS_2 \times S^2$ inside the $AdS_3 \times S^3$ set up by the black string with q units of worldvolume gauge field flux and the AdS_2 has a radius of curvature given by $l^2 = L^2(1+C^2)$ with C as in (3.13) with D5 and D3-branes replaced by D3 and D1-branes respectively and

$$k_N = g_{s,6d} \sqrt{Q_1 Q_5} \,. \tag{4.2}$$

4.2 6d black string with NSNS charges

For the system with F1 NS5-branes, the 3d radius of curvature only depends on Q_5 [9],

$$L^2 = l_s^2 Q_5 \tag{4.3}$$

and hence in this case

$$k_N = Q_5 \,. \tag{4.4}$$

Since only the fundamental strings end, the radius of curvature does not jump when crossing the AdS_2 brane. Instead, it is the 6d string coupling that jumps, since it is given by

$$\frac{1}{g_{s,6d}^2} = \frac{Q_5}{Q_1} \,. \tag{4.5}$$

Note that in the D3-D5 or D1/D5–D3 system, q, the number of branes ending, was bounded by N, the number of branes that were there to begin with, which in terms set the AdS curvature radius. Here q is bounded by Q_1, which only appears in the string coupling, not the curvature radius. Using (4.5) the bound reads

$$q \leq \frac{Q_5}{g_{s,6d}^2} \tag{4.6}$$

which looks like a non-perturbative bound.

This F1/N5–D3 system was recently studied in detail by Bachas and Petropoulos [5]. This system has the advantage that the near-horizon geometry has an exact CFT[1] description as a supersymmetric WZW model on $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ in which one can find the D-brane states. These turn out to be associated with an $AdS_2 \times S^2$ geometry, which is what we found as well.

To find the branes, one first finds the branes existing in the AdS_3 and S^3 independently, which correspond to conformal boundary states (separately) for the $\mathrm{SL}(2,\mathbb{R})$ and the SU(2). The D-branes for the full geometry can be obtained by putting these together, since the conformal boundary state for the full theory is found by tensoring together the conformal boundary states of each of the factors. Now the brane states of the S^3 are spherical $D2$-branes. They are stabilized at fixed radius by p units of magnetic flux for a system with p-branes, and have radius

$$r = L\ \sin(\pi\, p\, l_s^2/L^2)\,. \tag{4.7}$$

Bachas and Petropoulos also found the D-branes of the $\mathrm{SL}(2,\mathbb{R})$ WZW model, which are D-strings with AdS_2 geometry, where the AdS curvature is $l^2 = L^2(1+C^2)$ and they find that C is given by

$$C = qT_F/pT_D \,, \tag{4.8}$$

where T_F and T_D are the tension of the fundamental and D-strings respectively. q is the quantized F_{tx} along the AdS_2. p is the worldvolume gauge field flux through the S^2. Combining the D-strings and D_2-branes together gives a single D3-brane living in $AdS_3 \times S^3$ with associated $AdS_2 \times S^2$ geometry that preserves supersymmetry and conformal invariance. This D-brane carries the charge of a (p,q). string.

[1] That is instead of classical supergravity one can actually study tree level string theory on this background, removing the need for small curvatures, but still requiring small string coupling.

Note that their result agrees with our $M = 1$ result if we identify their q with our q and set $p = Q_5/2$ with an apropriate choice of the unknown constant of proportionality in (3.13). Our q is the number of strings ending on the D3. For the D1, q led to q units of magnetic flux through the S^2. In their case, q is the number of electric flux quanta turned on along the AdS_2. This may sound quite different, but it is actually once more just counting the number of strings ending, this time fundamental strings. But since they are turning on the S-dual electric field, as in [10], it is the F_{rt} components that get turned on instead of the $F_{\theta\phi}$ components.

The fact that $p = Q_5/2$ can easily be seen by looking at the S^2. Only for this particular value of p, the radius of the S^2 (4.7) is indeed L as we found. p counts the D1-brane charge of the wrapped D3-brane. Note that the D3 along 0123 and the NS5 along 056789 (where 6789 are the four internal directions) is dual to the NS5 012345 and D5 along 012789 of [11], that is they are "linked". What this means is that the presence of an NS5-brane induces half a unit of D1-brane charge on the D3-brane. So in our set-up, even though only F1 strings end on the D3-brane, the presence of Q_5 NS5-branes intersecting the D3-brane still induces $p = Q_5/2$ units of D1-brane charge on the worldvolume of the D3. In the near horizon geometry studied by [5] any other value of p lead to a supersymmetric configuration as well. It is unlikely that this freedom remains when including the asymptotic flat region.

4.3 The M2-brane ending on an M5-brane

Now let us focus on the M5-brane in the $AdS_4 \times S^7$ geometry set up by N M2-branes. For the intersecting case, $q = 0$, the story goes through as above. We end up with a $C = 0$ AdS_3 inside the AdS_4 times an equatorial S^3 inside the S^7. Once we let q of the M2-branes end, the story changes slightly. We still turn on q units of the wordvolume 3-form flux through the S^3. However this time the brane is codimension 4 and so the bending goes like C/r^2, and hence

$$x = \frac{C}{u^2} \tag{4.9}$$

seems to describe the embedding inside the AdS_4. It is possible that the $1/r^2$ behavior is only adequate far away from the intersection and that in the M2 near-horizon region the M5-brane bending actually is $1/r$ and then turns over. This kind of behavior seems to be suggested by the analysis of [12], where it is found that a D-string ending on any higher dimensional D-brane leads to a $1/r$ bending close to the brane, while only far from the intersection one finds the naive bending.

4.4 Generalities

We see that with branes ending on branes, there is in general a spike where the branes are pulled; for example, if D3-branes end on the 5-brane, the 5-branes grow a spike where they are pulled by the D3-branes [8]. This spike could signal that the boundary conditions actually break conformal invariance; that is while the bulk β function still vanishes by construction, there might be a non-zero boundary β function. In the case of intersecting branes, one has the same number of branes on the other brane from either side and there is no bending and the boundary conditions clearly preserve conformal invariance. In order to preserve conformal invariance with branes ending, the spike has to be a $1/r$ spike.

Even though on the probe level the 4d and 2d set-ups seem to be very similar in character, it is clear that the physics is somewhat different. While in the 4d case in the $l_s \to 0$ decoupling limit the 6d modes decouple, leaving us with a 4d field theory with a 3d defect, in the black string case the decoupling limit for the 2d modes also leaves the D3-brane gauge coupling finite. Probably in the gravity interpretation, this is related to the observation of [13] that lower dimensional brane metrics never completely localize but are always smeared.

5. The Born-Infeld calculation

As a confirmation of the results derived in the last subsection and to determine the constant α in (3.13), we study the worldvolume action of a probe D5-brane in the $AdS_5 \times S^5$ background set up by N D3-branes. The DBI part of the AdS action only gets a contribution from the induced metric and is competing with the contribution from the WZ term, which couples the q units of worldvolume flux through the sphere to the background RR 4-form turned on along the AdS_4 due to the non-vanishing RR 5-form flux turned on in the AdS_5. A similar probe calculation was done for the F1/NS5 system in [5]. There the mechanism was quite different, since it is the NSNS 2-form that is turned on, which contributes via the DBI term of the action. Their probe calculation confirmed the results from the WZW calculation which we reviewed above.

So let's start with the $AdS_5 \times S^5$ of radius $L^2 = k_N l_s^2$. As an ansatz for the embedding we assume the D-brane wraps an S^2 inside the S^5, but allow a representative 3rd angle ψ to be arbitrary. The induced metric on the S^2 is hence

$$ds^2 = r^2(d\theta^2 + \sin^2(\theta)d\phi^2)\,, \tag{5.1}$$

where the radius of the S^2 is given by

$$r = L\sin(\psi)\,. \tag{5.2}$$

In the AdS_5 space (v, x, y, z, t) we take the string to be static and be embedded by

$$v = v(x) \tag{5.3}$$

so that the induced metric on this 4d part of the worldvolume becomes

$$ds^2 = L^2 v(x)^2(dt^2 - dy^2 - dz^2) - L^2\left(v(x)^2 + \frac{v'(x)^2}{v(x)^2}\right)dx^2\,. \tag{5.4}$$

In addition there are q units of worldvolume flux through the S^2

$$\int_{S^2} F = 2\pi q \tag{5.5}$$

and since no B-field is turned on, $\mathcal{F} = 2\pi l_s^2 F$. The $AdS_5 \times S^5$ background in addition has N units of 5-form flux turned on. We take the corresponding 4-form vector potential to be given by

$$C = L^4 v^4 dx \wedge dy \wedge dz \wedge dt\,. \tag{5.6}$$

The action of a D5 probe brane is obtained from the DBI lagrangian

$$\mathcal{L} = -T_{D5}\sqrt{\det(g + 2\pi l_s^2 F)} \tag{5.7}$$

and the WZ term coupling C to F. First let us analyze the DBI contribution. Since F is only turned on along the sphere, $g + \mathcal{F}$ is block diagonal and the determinant factorizes,

$$\mathcal{L} = -T_{D5}\,\mathcal{L}_{S^2} \times \mathcal{L}_{AdS_4}\,. \tag{5.8}$$

Let's first look at the S^2 part of the DBI

$$\mathcal{L}_{S^2} = \int_{S^2} \sqrt{\det(g+\mathcal{F})} = 4\pi L^2 \sqrt{\sin^4(\psi) + \frac{\pi^2 q^2}{k_N^2}}\,. \tag{5.9}$$

Since this is the sum of two positive contributions, clearly it can be minimized by $\psi = 0$, which would correspond to a collapsed sphere, while $\sin(\psi) = 1$ is a maximum. Since we found before that the S^2 actually has maximum size we proceed with putting $\sin(\psi) = 1$. Since the remaining part of spacetime is an AdS_4, we don't need to be bothered by the tachyonic mode corresponding to fluctuations in ψ since they don't violate the Breitenlohner-Freedman bound [7], as we will show later. For the AdS_4 part we just get the contribution from the induced metric

$$\mathcal{L}_{AdS_4} = \sqrt{\det(g)} = L^4 v^2 \sqrt{v^4 + (v')^2}\,. \tag{5.10}$$

Last but not least we have to include the WZ part, whose relevant piece coupling the RR 4-form to the worldvolume field strength reads

$$\mathcal{L}_{WZ} = T_{D5}\, 2\pi l_s^2\, F \wedge C\,. \tag{5.11}$$

Using (5.6) and performing the integral over the S^2 using (5.5) this evaluates to

$$T_{D5}\, 4\pi^2 l_s^2\, q L^4 v^4$$

and the effective 4d lagrangian reads

$$\mathcal{L}_{4d} = -T_{D5} 4\pi L^4 \left(L^2 \sqrt{1 + \frac{\pi^2 q^2}{k_N^2}} v^2 \sqrt{v^4 + (v')^2} - \pi l_s^2 q\, v^4 \right)\,. \tag{5.12}$$

Clearly, for $q = 0$, we have AdS space with the four dimensional AdS curvature inherited from the five dimensional one. For nonzero q, we still find an AdS solution, where the two AdS curvature scales are qseparated: amazingly enough the equations of motion derived from this lagrangian

$$2L^2\left(v\sqrt{v^4+(v')^2}+\frac{v^5}{\sqrt{v^4+(v')^2}}\right)-\frac{4\pi\, l_s^2 v^3 q}{\sqrt{1+\pi^2q^2/k_N^2}}=L^2\frac{d}{dx}\frac{v'v^2}{\sqrt{v^4+(v')^2}} \tag{5.13}$$

are indeed satisfied for

$$v=\frac{C}{x} \tag{5.14}$$

with

$$C=\frac{\pi q}{k_N} \tag{5.15}$$

in perfect agreement with (3.13) for $M = 1$. Using the descent relation for D-brane tensions

$$T_{D(p-2)}=T_{\mathrm{D}p}\,4\pi^2 l_s^2 \tag{5.16}$$

this fixes α in (3.13) in this case to be $1/4\pi$.

The same calculation goes through almost unchanged for the D3-brane in the D1/D5 system. The effective lagrangian for the non-compact 2d part in AdS_3 similarly to above becomes

$$\mathcal{L}_{2d}=-T_{D3}4\pi L^2\left(L^2\sqrt{1+\frac{\pi^2q^2}{k_N^2}}\sqrt{v^4+(v')^2}-\pi l_s^2 q v^2\right) \tag{5.17}$$

and one can once more verify that the equations of motion have the desired AdS_2 solution.

Last but not least let's return to the issue of stability. There seems to be a potential tachyonic instability with the assumption that $\theta = \pi/2$, since it seems at first glance that the brane would not be stable on the equator, but would slip off. However, this is not the case. The scalar is stabilized because of the surrounding AdS space. In fact, it is easy to see the scalar satisfies the BF bound.

Let us consider a perturbation in ψ so that $\psi = \pi/2 + \delta(t)$. We evaluate $\det(g + \mathcal{F})$, the Born-Infeld action, where $\mathcal{F}$ is the flux

$$\mathcal{F}=q\pi\, l_s\sin\theta d\theta d\phi\,, \tag{5.18}$$

and the metric is of the form

$$ds^2=-L^2\sin^2\psi\left(d\theta^2+\sin^2\theta d\Phi^2\right)+L^2\frac{C^2}{x^2}dt^2-\cdots \tag{5.19}$$

where we only kept the time component along the AdS_4 direction, which is sufficient to determine the normalization of the kinetic term. With this we obtain

$$\sqrt{\det(g+\mathcal{F})} = 4\pi L^3 \sqrt{1+\frac{\pi^2 q^2}{k_N^2}} \left(\frac{x}{C}\dot{\delta}^2 + \frac{C}{x}\frac{\delta^2}{1+\pi^2 q^2/k_N^2} \right)$$
$$\sim \frac{x^2}{L^2C^2}\dot{\delta}^2 + \frac{\delta^2/L^2}{1+\pi^2q^2/k_N^2} \,. \tag{5.20}$$

Recognizing x^2/L^2C^2 as g^{00} and using our result that

$$l^2 = L^2\left(1+\frac{\pi^2q^2}{k_N^2}\right)$$

we see that δ describes an excitation of

$$m^2 = -\frac{1}{l^2} \tag{5.21}$$

which is above the bound of $-9/4l^2$ [7] independent of q.

6. Beyond the probe approximation

6.1 Parameters controlling the backreaction

So far we have neglected the backreaction of the D5-brane on the background geometry of the near horizon D3-branes, or the corresponding analogs in other dimensions. This is important if we want to understand how the interplay of boundary versus bulk modes in the CFT is reflected in the spectrum of the gravity fluctuations. Since at the moment the full localized supergravity solution is not known, we can't present an analytic formula for the exact warpfactor. Given our knowledge about the symmetries and asymptotic behavior it shouldn't be too difficult to construct it along the lines of [14, 15]. At least as interesting would be to construct the 5d supergravity solution that would lift to our solution. A future goal it to construct the solution as an AdS domain wall in $\mathcal{N}=8$ gauged supergravity in 5d, since effectively 5 dimensions got compactified in the D3-brane throat. The internal geometry of the S^5 will be reflected in a flow of the 5 SO(3) $\times$ SO(3) invariant scalars in the theory.

Let us briefly consider some general scaling arguments. In the 5d string frame the Einstein-Hilbert term is given by

$$\frac{L^5}{g_s^2}R \tag{6.1}$$

while the tension of M D5-branes wrapping the S^2 of radius L^2 with q units of flux is given by

$$4\pi\, M\, T_{D5}\left(L^2\sqrt{1+\frac{\pi^2q^2}{k_N^2}}\right) \tag{6.2}$$

where we need to recall that $T_{D5} \sim 1/g_s$. So the jump in extrinsic curvature is proportional to

$$M g_s \sqrt{1 + \frac{\pi^2 q^2}{k_N^2}} \,. \tag{6.3}$$

First note that for $g_s \to 0$, $g_s N$ fixed, but $M g_s q$, $g_s M \to 0$, the backreaction can be neglected. In this regime the probe calculation is accurate; we get a very flat AdS_4 (note that it is q and not $g_s q$ that controls the curvature of the AdS_4), but of course we won't get localized modes. For this we do need to consider a parameter regime with a significant backreaction.

There are two interesting cases, both of which have a free parameter that permits us to separate the four and five dimensional curvature scales. In this paper, we consider large $g_s q$ with finite M and vanishing $g_s M$. As the probe calculation of the previous section showed, by adjusting q at $g_s q = 0$ we can start with an arbitrarily flat AdS_4 and then continously turn on the backreaction. Since $g_s M \to 0$ we won't source the dilaton or the 2-form. The 5-brane carrying 3-brane charge gets replaced with q smeared D3-branes. The second case is considered in ref. [16] and is $q = 0$ and finite $g_s M$, and we expect can localize gravity.

6.2 Possible behavior of the warp factor

Before we discuss the gravity solutions with the backreaction included, let us pause and discuss the two possible behaviors of the warpfactor and what this would mean in terms of the dual CFT. For this we study as a toy model a thin, charged 3-brane in AdS_5. That is we allow the AdS_5 radius to jump from $k_N^2 l_s^2$ on the left to $k_{N-q}^2 l_s^2$ on the right.

Turning on the backreaction, the warpfactor has to jump at the brane. In a Z_2 symmetric set-up as in [17, 2], this has to be "up-down". That is, if the warp factor initially has positive slope, on the other side of the positive tension brane it has negative slope. For the asymmetric case with the additional jump in background cosmological constant, there are in principle two possibilities, which we refer to as "up-slower up", figure 2, and "up-down", figure 3. For a thin brane of tension λ the warp factors read

$$e^A = \begin{cases} \sqrt{|\Lambda|} L_L \cosh(b_L + k_L r) & \text{for } r < 0 \\ \sqrt{|\Lambda|} L_R \cosh(b_R + k_R r) & \text{for } r > 0 \end{cases} \tag{6.4}$$

where $|k_{L,R}| = 1/L_{L,R}$, $k_L > 0$ and the sign of k_R distinguishes the two possibilities. The parameters $b_{L,R}$ are determined from the jump conditions

$$L_L \cosh(b_L) = L_R \cosh(b_R) \,, \qquad \lambda = \frac{3}{2}(k_L \tanh(b_L) - k_R \tanh(b_R)) \tag{6.5}$$

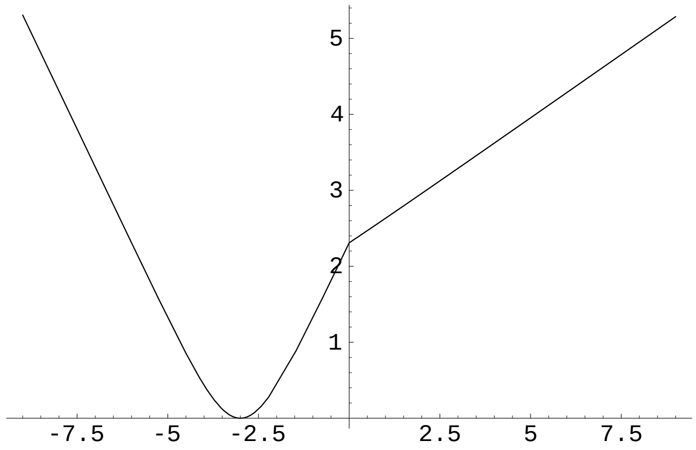

Figure 2: "Up-slower up" AdS warpfactor for $L_L = 1$ and $L_R = 3$.

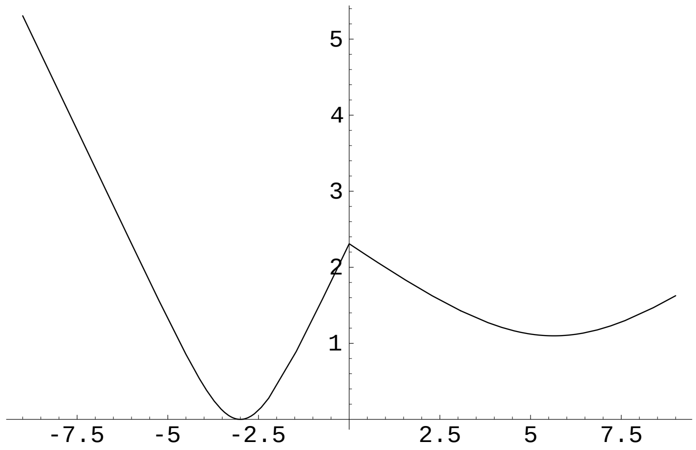

Figure 3: "Up-down" AdS warpfactor for $L_L = 1$ and $L_R = 3$.

and the 4d cosmological constant of the model is

$$\Lambda = \frac{1}{9\lambda^2}\left[\left(\frac{9}{4}(k_L - k_R)^2 - \lambda^2\right)\left(\frac{9}{4}(k_L + k_R)^2 - \lambda^2\right)\right]. \tag{6.6}$$

The difference between the two in terms of the gravity fluctuations is that the "up-down" warpfactor has a mode that approximately gives rise to a 4d Newton's law, while the "up-slower up" warpfactor always looks five dimensional. In order to see this, consider for simplicity the "up - slower up" Minkowski set-up, as it would for example arise on the Higgs branch of the $\mathcal{N} = 4$ SYM, where SU$(N+1)$ is broken to SU(N). The additional D3-brane that is separated from the stack of N D3-branes in the AdS_5 geometry appears as a domain wall, across which the background curvature jumps.

Figure 5 displays the volcano potential in this case. Obviously this does not lead to localized gravity. The zero mode is not normalizable, since the warp factor diverges on the right. However, as in the AdS case, this would lead to 4d physics if there

JHEP06(2001)063

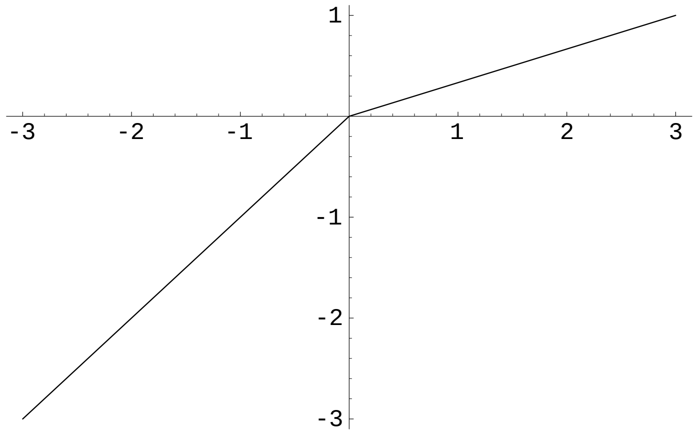

Figure 4: "Up-slower up" Minkowski warpfactor for $L_L = 1$ and $L_R = 3$.

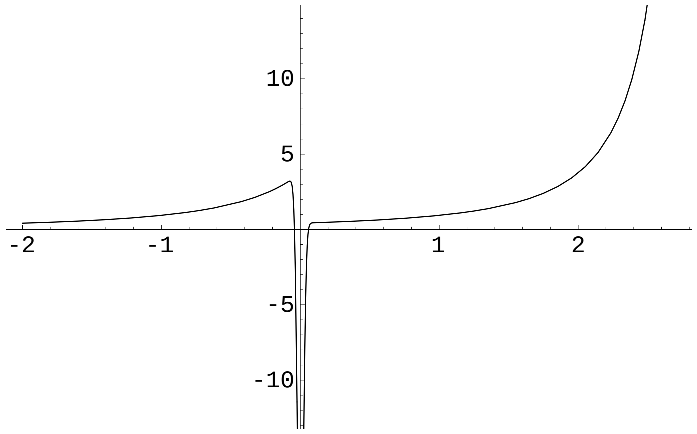

Figure 5: Volcano potential associated to the "up-slower up" Minkowski warpfactor for $L_L = 1$ and $L_R = 3$.

were a bound state (or in the Minkowski case a resonance) that dominated over the KK modes. This clearly does not happen in this case, where on the right hand side, the bound state amplitude would be small, while the KK modes are unsuppressed, though coming in from the left, they are surpressed by the usual barrier. The barrier on the right is set by $1/L_R^2$ (the onset of the potential on the right of the delta), which is also the natural mass scale for the resonances, so there is no regime in which the 5d modes can be separated from a "4d" resonance. In the AdS_4 warping all modes are normalizable, but again to have the light mode separated from the 5d KK modes, we need an "up-down" warpfactor.

It is interesting to understand the distinction between the localizing warp factors in terms of the dual CFT. After all, both situations correspond to an AdS brane (in the geometries we study) with the dual CFT living on half the boundary, as previously described. We have argued that in the "up-down" situation, we can get a 4d Newton law, while in "up-slower up" Newton's law essentially stays 5d. In the dual CFT this has to be reflected in a property of the 2-point function of the stress-energy

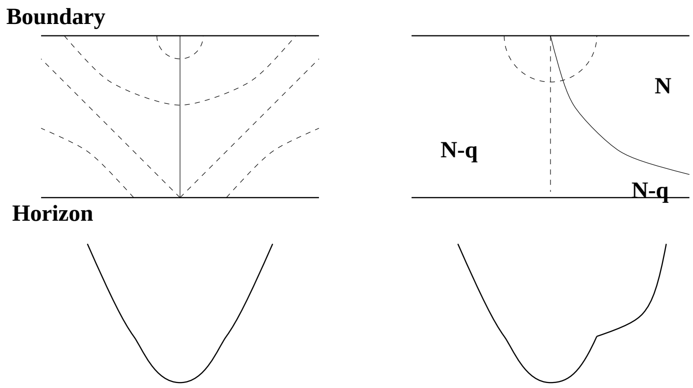

Figure 6: The $q = 0$ and finite q D5-branes in the D3-brane near horizon geometry. The dashed lines show the lines parametrized by the r coordinate of the AdS_4 slicing. The corresponding warp factors as a function of r are also displayed.

tensor. The bulk 2-point function goes like $1/|x|^8$, consistent with coupling to the 5d graviton. Since the presence of the boundary broke the symmetry to SO$(3, 2)$ already, we can have a 3d CFT living on the codimension one defect coupled to the 4d CFT. Its stress energy tensor to leading order has $1/|x|^6$ correlations and hence naturally couples to a 4d graviton. So the difference between "up-down" and "up-slower up" can be interpreted as whether we get $1/|x|^8$ or at least approximately $1/|x|^6$ correlators. For latter it is necessary but not sufficient to have a dynamical 3d CFT on the defect.

6.3 Turning on the backreaction

Let us first discuss the case with finite M, so that $g_s M \to 0$ and $g_s q$ finite. In this case the AdS_4 can be very flat already in the probe limit. In addition only 3-brane fields are turned on, the source terms for dilaton and 2-form vanish, so that the analysis of the supergravity solution should simplify. However we will argue that there are several reasons why this warp factor has to be "up-slower up".

First consider small $g_s q$. Up-down is the only choice that continously reproduces the probe result as $g_s q$ goes to zero. In the probe limit the warpfactor is just the pure AdS_5 warpfactor

$$e^A(r) = \sqrt{|\Lambda|}L \cosh(r/L - b) \tag{6.7}$$

with the brane at $r = 0$ sitting at a warpfactor $\sqrt{|\Lambda|}L \cosh(b)$, which is large for large q. Now we want to turn on the backreaction. The warpfactor becomes different on both sides and is characterized by b_L and b_R respectively. b gives the distance between brane and turnaround point. The sign of b tells us if the turnaround is to the left or to the right of the brane. Writing the pure AdS warpfactor (6.7) from the probe limit as a left and a right warpfactor, we have $b_L = b_R = b$. There is only one turnaround point and it is to the left of the brane. Now as we smoothly turn on $g_s q$, b_L and b_R will separate proportional to $g_s q$. For an "up down" warpfactor we need a second turnaround point to the right of the brane, that is we need b_R to change sign. Since we start out with large b, changing the sign is not a small peturbation. This argument rules out "up-down" for small $g_s q$, we will now present two more arguments that suggest that this is still the case once we turn on a significant backreaction.

A picture of this situation as seen in the AdS_5 Poincaré patch (as it arises in the near horizon limit of the D3-branes) is displayed in figure 6. Far to the right (for large x_6) the 5-brane is almost horizontal, so that locally we just get the $\mathcal{N} = 4$ SYM on the Higgs branch, breaking SU(N) to SU(N) $\times$ SU($N - q$).[2] For this case the full 10d metric is known, since it is just a multi-centered D3-brane metric. By inspection one finds that the warp factor in this case is "up-slower up", e.g. in 5d language, an SO(6) invariant configuration of D3-branes gives rise to the warpfactor of figure 4. By continuity we expect this then to be the case everywhere along the brane, not just at large x_6.

Last but not least, the dual CFT also supports the point of view that the $g_s M = 0$, $g_s q$ finite configuration does not localize gravity. Above we argued that 3d CFT living on the boundary defect is required. For a single D5-brane intersecting the N D3-branes, the only 3d degrees of freedom are the hypers from 3-5 strings, which should flow in the IR to a theory containing no dynamical degrees of freedom.

In another paper, we consider the second possibility, that is turning on $g_s M$ while leaving $q = 0$. In this case we have an additional Z_2 symmetry, so that "up-slower up" is not a possibility. The brane starts out sitting at the turnaround point of the warpfactor in the probe limit. Including its backreaction by turning on $g_s M$ the warp factor will this time grow a little "up-down" spike in the center, as we discuss in more detail in [16]. In this case, all three of the above arguments fail and we expect the theory does in fact have a parameter regime in which gravity is localized.

7. Conclusions

In this paper we have given a stringy realization of the duality conjecture in [2] between conformal field theories on a manifold with boundary and an AdS brane inside AdS. We showed that in brane configurations which realize such supersymmetric CFTs with codimension one conformal defects, the higher dimensional brane in the near horizon geometry of the lower dimensional brane spans a worldvolume of the

[2]This picture actually seems to capture quite acurately what is going on in the dual SYM. In order to get the situation with SU(N) on one side of the defect and SU($N - q$) on the other side of the defect, one can start with SU(N) everywhere and turn on a VEV for one of the impurity hypermultiplets from the 3-5 strings, corresponding to moving off q D3-branes on the left [18]. The vacuum solution in the presence of this VEV seems to demand that on the right now the bulk Higgs fields have to develop a position dependend $1/x_6$ VEV, so that the SU(N) on the right gets broken to SU($N - q$) as well and is only restored asymptotically. This is how string theory realizes the interaction of two conformal field theories with different central charge across a boundary. We like to thank E. Martinec for a useful discussion on this point.

form AdS times sphere. We gave some general arguments of how the 3 numbers M, N and q for q out of N lower dimensional branes ending on M higher dimensional branes, control the backreaction and hence the ratio of 4d and 5d curvature scales (refering to the D3 D5 system to which we devoted most of our attention) and how the interplay of 4d and 5d modes in the gravity reflects the interaction between bulk and boundary modes in the CFT.

Acknowledgments

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